STUDENT NUMBER ____________________

NAME: ....

Trial HSC Mathematics

Extension 1 - 2013

Time Allowed - 2 hours + 5minutes reading

Instructions: Calculators may be used in any parts of the task. For 1 Mark Questions, the correct answer is sufficient to receive full marks. For Questions worth more than 1 Mark, necessary working MUST be shown to receive full marks.

Multiple Choice	/10
Question 11	/15
Question 12	/15
Question 13	/15
Question 14	/15
Total	/70

Fill in the correct answer on the answer sheet - Questions 1 - 10 are worth 1 mark each

1. A parabola has the parametric equations $x = 12t$ and $y = -6t^2$.

The coordinates of the focus are

A $(-6, 0)$ B $(0, -6)$

C $(6, 0)$ D $(0, 6)$

2. If $t = tan\frac{1}{2}\theta$ then $\frac{1+t^2}{1-t^2}$ is equivalent to

A $cot\theta$ B $cos\theta$

C $sec\theta$ D $cosec\theta$

3. $\frac{\sin(360°-A)}{\sin(90°-A)} =$

A -1 B $tanA$

C 1 D $-tanA$

4. What is the equation of the chord of contact of tangents from $(-2, -1)$ to $x^2 = 4y$

A $x + y - 1 = 0$ B $x + y + 4 = 0$

C $x + 2y - 1 = 0$ D $x + 2y + 4 = 0$

5. Which expression is equal to $\int sin^2 3x\, dx$?

A $\frac{1}{2}\left(x - \frac{1}{3}sin3x\right) + C$ B $\frac{1}{2}\left(x + \frac{1}{3}sin3x\right) + C$

C $\frac{1}{2}\left(x - \frac{1}{6}sin6x\right) + C$ D $\frac{1}{2}\left(x + \frac{1}{6}sin6x\right) + C$

6. How many arrangements of the word GEOMETRY are possible if the T and R are to be together in any order?

A $2 \times 6!$

B $2 \times 7!$

C $7!$

D $\frac{1}{2} \times 7!$

7. Which of the following is an expression for $\int cos^2x \sin x \; dx$?

A $2 \cos x \sin x + c$

B $cos^3x + c$

C $\frac{1}{3} cos^3x + c$

D $-\frac{1}{3} cos^3x + c$

8.. What is the acute angle between the lines $y = x + 2$ and $y = 2x - 1$?

A $18^\circ 26'$

B $19^\circ 28'$

C $70^\circ 32'$

D $71^\circ 34'$

9. If $f(x) = e^{x+2}$ what is the inverse function $f^{-1}(x)$?

A $f^{-1}(x) = e^{y-2}$

B $f^{-1}(x) = e^{y+2}$

C $f^{-1}(x) = log_e x - 2$

D $f^{-1}(x) = log_e x + 2$

10. What is the derivative of $y = \cos^{-1}\left(\frac{1}{x}\right)$ with respect to x?

A $\frac{-1}{\sqrt{x^2-1}}$

B $\frac{-1}{x\sqrt{x^2-1}}$

C $\frac{1}{\sqrt{x^2-1}}$

D $\frac{1}{x\sqrt{x^2-1}}$

Question 11 15 Marks **(Begin a new sheet of paper)** **Marks**

a) The point P divides the interval AB joining A $(-4, -3)$ and B $(1, 5)$ externally in the ratio 3:2. Find the coordinates of P. **2**

b) At a dinner party, the host, hostess and their six guests sit at a round table. In how many ways can they be arranged if the host and hostess are separated? **2**

c) A stone is thrown from the top of a 15m cliff with an initial velocity of $26ms^{-1}$ at an angle of projection equal to $\tan^{-1}\left(\frac{5}{12}\right)$ above the horizontal.

(i) Taking $g = -10$, derive equations for x and y in terms of t. **3**

(ii) Calculate the time when the stone will reach the ground. **2**

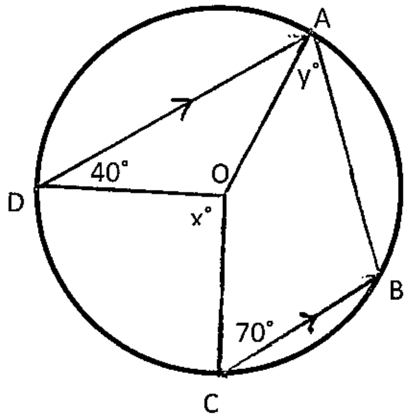
d) Solve $\frac{x^2-4}{2x} < 0$ and graph your solution on a number line. **3**

e) Differentiate $y = x\sin^{-1}x + \sqrt{1-x^2}$ **3**

Hence evaluate $\int_0^1 \sin^{-1}x\,dx$.

Question 12 15 Marks **(Begin a new sheet of paper)** **Marks**

a) O is the centre of the circle and AD is parallel to BC.
Find the values of x and y. 3



b) The acceleration of a particle is defined in terms of its position by the equation $a = 2x + 4$. If $v = 5$ when $x = 2$, what is the velocity when $x = 4$? 3

c) The normal to a parabola $x^2 = 4ay$ at a point $P\ (2ap, ap^2)$ on it cuts the axis of the parabola at G.

(i) Show that the normal has equation $x + py = 2ap + ap^3$. 2

(ii) Find the coordinates of G 1

(iii) Find the equation of the locus of the mid-point M of PG 2

d) Cane sugar, when placed in water, converts to dextrose at a rate proportional to the amount of unconverted material remaining. That is, if M grams is the amount of material converted after t minutes, then

$$\frac{dM}{dT} = k(S - M)$$ where S grams is the initial amount of cane sugar and k is a constant.

(i) Show that $M = S + Ae^{-kt}$ satisfies the equation, where A is constant. 1

(ii) If a certain amount of cane sugar is placed in water at time $t = 0$ and 40% of it has been converted after 10 minutes, show $k = \frac{1}{10} log_e \frac{5}{3}$. 2

Question 13 15 Marks **(Begin a new sheet of paper)** **Marks**

a) A particle is oscillating in simple harmonic motion about a fixed point O on a straight line. At time t seconds its displacement x metres from O satisfies the equation

$$\frac{d^2x}{dt^2} = -4x$$

(i) Show that $x = a\cos(2t + \beta)$ is a possible equation of motion for this particle, where a and β are constants and $a > 0$. State the period. **2**

(ii) The particle is observed at time $t = 0$ to have a velocity of 2 metres per second and a displacement from the origin of 4 metres. Show that the amplitude of the oscillation is $\sqrt{17}$ metres. **2**

(iii) Show that β is approximately -0.245 correct to 3 decimal places. **1**

(iv) Give the general solutions to the times when the particle is back at its starting point. **2**

(v) Find the time at which the particle first returns to its starting point. **1**

b) By using the substitution $u = \sqrt{x+1}$ find $\int_3^8 \frac{x-1}{\sqrt{x+1}}$ dx **3**

c) Use the remainder theorem to find one factor of $x(x+1) - a(a+1)$. By division, or otherwise, find the other factor. **2**

d) It is known that the equation $e^{-x^2} - 5x^2 - 0.99 = 0$ has a positive root close to the origin. Attempt to find this root using Newton's method, starting with a first approximation of $x_0 = 0$. Explain why this method fails. **2**

Question 14   15 Marks   **(Begin a new sheet of paper)** **Marks**

a) Prove by induction that

$$\frac{1}{x^n(x-1)} = \frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2} - \quad . . . - \frac{1}{x^n}$$ 4

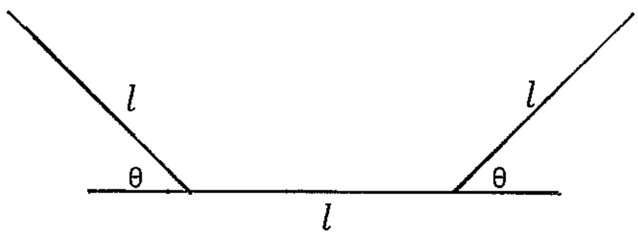
for all positive integers n where $x \neq 0$ or 1.
(Hint: When $n = 1$, RHS contains two terms.)

b) Let α, β and γ be the roots of $3x^3 + 8x^2 - 1 = 0$.

(i) State the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$, and $\alpha\beta\gamma$. 1

(ii) Find the value of $\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\gamma}\right)\left(\gamma + \frac{1}{\alpha}\right)$ 3

c) An irrigation channel is to have a cross-section in the shape of a trapezium. The bottom and sides are each l metres long. The sides of the channel make an angle $\theta \leq \frac{\pi}{2}$ with the horizontal.



(i) Show that the area of cross-section of the channel, as a function of θ, is

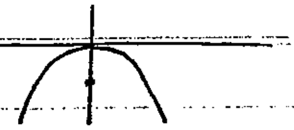
$A = l^2 \sin\theta\,(1 + \cos\theta)$ (l is a constant). 3

(ii) For what angle θ is the area of the cross-section a maximum? 4

End of Test

Solutions to 2013 Ext 1 Trial.

1. B $(0,-6)$



B

2. $\cos\theta = \dfrac{1-t^2}{1+t^2}$ $\therefore$ $\dfrac{1+t^2}{1-t^2} = \sec\theta$ C

3. $\dfrac{\sin(360-A)}{\sin(90-A)} = -\dfrac{\sin A}{\cos A} = -\tan A$ D

4. $x x_0 = 2a(y+y_0)$ $a = 1$

$-2x = 2(y-1)$

$0 = 2x+2y-2$ A

$0 = x+y-1$

5. $\cos 2\theta = 1 - 2\sin^2\theta$

$2\sin^2\theta = 1 - \cos 2\theta \implies \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

$\int \sin^2 3x\,dx = \frac{1}{2}\int 1 - \cos 6x\,dx$

$= \frac{1}{2}\left(x - \frac{1}{6}\sin 6x\right)$ C

6. Bind them together $2!$ ways

Arrange 7 things $7!$

2 are alike $\therefore$ $\dfrac{2!\times 7!}{2!} = 7!$ C

7. $\int \cos^2 x \sin x\,dx = \int -\cos^2 x(-\sin x)\,dx$

$= -\dfrac{\cos^3 x}{3} + C$ D

8. $m_1 = 1$ $m_2 = 2$ $\tan\theta = \left|\dfrac{1-2}{1+2}\right|$

$\tan\theta = \frac{1}{3}$ $\theta = 18^\circ 26'$ A

9. $y = e^{x+2}$

$x = e^{y+2}$ C

$y+2 = \ln x$

$y = \ln x - 2$

10 $y = \cos^{-1}\frac{1}{x}$ $\quad y' = \frac{-1}{\sqrt{1-\frac{1}{x^2}}} \cdot -x^{-2}$

$y' = \frac{1}{x^2\sqrt{1-\frac{1}{x^2}}} = \frac{1}{x\sqrt{x^2-1}}$ 1)

(Note: $\frac{1}{x} = \cos y \Rightarrow x = \frac{1}{\cos y} = \sec y$

$\therefore y = \sec^{-1} x$. We have differentiated $y = \sec^{-1} x$)

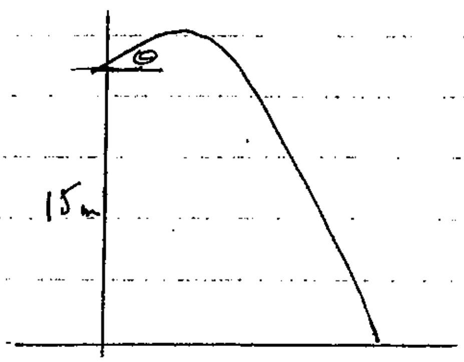
Q11 a) $x = \frac{kx_2 + lx_1}{k+l}$ $\quad y = \frac{ky_2 + ly_1}{k+l}$ $\quad \frac{k}{l} = \frac{3}{-2}$

$x = \frac{3\times 1 + -2\times -4}{3-2}$ $\quad y = \frac{3\times 5 + -2\times -3}{3-2}$

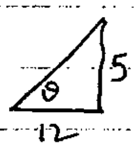
$x = 11$ $\quad y = 21$ $\quad P(11, 21)$

b) 8 people. Put host + hostess together 2 ways
Arrange other 6 $\quad 6!$ ways
$\therefore 2\times 6!$ with host + hostess together.
$\therefore$ Separated $7! - 2\times 6! = 3600$ ways

c) $V = 26$ $\quad \theta = \tan^{-1}\frac{5}{12}$



Initially



$\dot{y} = \frac{5}{13}\times 26 = 10$

$\dot{x} = \frac{12}{13}\times 26 = 24$

$\therefore \ddot{x} = 0$ $\quad \ddot{y} = -10$

$\dot{x} = 24$ $\quad \dot{y} = -10t + 10$

$x = 24t$ $\quad y = -5t^2 + 10t + 15$

ii) $y = 0$ $\quad -5t^2 + 10t + 15 = 0$

$t^2 - 2t - 3 = 0$

$(t-3)(t+1) = 0$

$t = 3$ or -1

$\therefore$ Stone hits ground after 3 seconds.

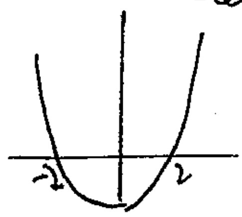
11 d) $\dfrac{x^2-4}{2x} < 0$

Consider $x > 0$

Solve $x^2 - 4 < 0$

$(x-2)(x+2) < 0$

$0 < x < 2$



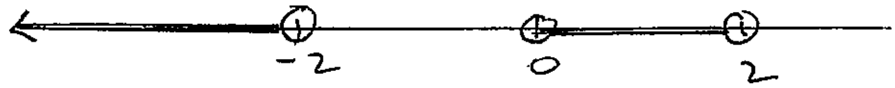
Consider $x < 0$

Solve $x^2 - 4 > 0$

$(x-2)(x+2) > 0$

$x < -2$

$\therefore \quad 0 < x < 2 \quad$ or $\quad x < -2$



e) $y = x \sin^{-1} x + \sqrt{1-x^2}$

$y' = x \cdot \dfrac{1}{\sqrt{1-x^2}} + \sin^{-1}x \cdot 1 + \frac{1}{2}(1-x^2)^{-1/2} \cdot -2x$

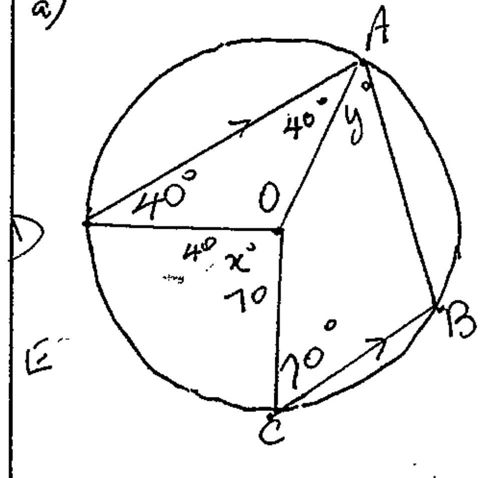
$= \dfrac{x}{\sqrt{1-x^2}} + \sin^{-1}x - \dfrac{x}{\sqrt{1-x^2}} = \sin^{-1}x$

$\therefore \displaystyle\int_0^1 \sin^{-1}x\,dx = \left[x \sin^{-1}x + \sqrt{1-x^2}\right]_0^1$

$= \sin^{-1}1 + 0 - 0 - \sqrt{1}$

$= \dfrac{\pi}{2} - 1$

Q 12 a)



$\angle DAO = 40°$ (isosceles $\triangle$ $OD = OA$)

Draw $OE \parallel AD$

$\therefore \angle DOE = 40°$ (alternate $\angle$'s

$\angle EOC = 70°$ (alternate $\angle$'s in $\parallel$ lines)

$\therefore x = 110°$

$\angle AOD = 100°$ (angle sum of $\triangle$)

$\therefore \angle ABC = \frac{1}{2}(100 + 110°)$

$\angle ABC = 105°$ (Angle at centre is twice $\angle$ at circumference standing on major arc AC.

Also $\angle AOC = 360 - (100 + 110) = 150°$

$\therefore \quad y = 360 - (70 + 150 + 105)$ angle sum of quad.

$y = 35°$

12b)

$$\ddot{x} = 2x + 4$$

$$\ddot{x} = \frac{d}{dx}\left(\tfrac{1}{2}v^2\right)$$

$$\therefore \tfrac{1}{2}v^2 = \int 2x + 4 \quad dx$$

$$\tfrac{1}{2}v^2 = x^2 + 4x + c_1$$

$$v^2 = 2x^2 + 8x + c$$

$v = 5,\ x = 2$

$$\therefore 25 = 8 + 16 + c$$

$$1 = c$$

$$v^2 = 2x^2 + 8x + 1$$

when $x = 4$,

$$v^2 = 32 + 32 + 1$$

$$v^2 = 65$$

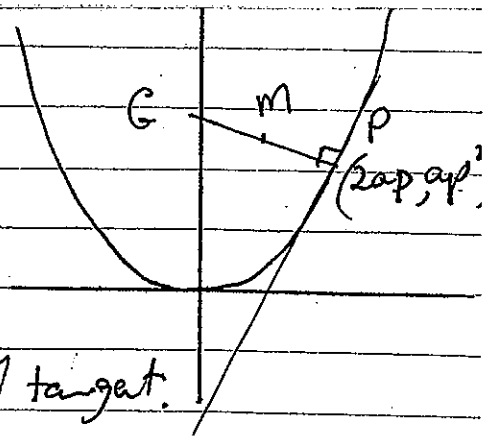
$$v = \pm\sqrt{65}$$

But $v > 0$ when $x = 2$, & $a > 0$ when $x = 2$

$$\therefore v = \sqrt{65}$$

c) $x = 2ap$

$y = ap^2$



$$\frac{dy}{dx} = \frac{\frac{dy}{dp}}{\frac{dx}{dp}}$$

$$= \frac{2ap}{2a} = p = \text{Gradient of tangent.}$$

Gradient of normal is $-\frac{1}{p}$

$\therefore$ Normal is $y - ap^2 = -\frac{1}{p}(x - 2ap)$

$$py - ap^3 = -x + 2ap$$

$$x + py = 2ap + ap^3$$

Provided $p \neq 0$, this cuts $x = 0$ at G

$$py = 2ap + ap^3$$

$$y = 2a + ap^2$$

$$\therefore G \text{ is } (0,\ 2a + ap^2)$$

M is $\quad x = \dfrac{2ap+0}{2} = ap$

$y = \dfrac{ap^2 + 2a + ap^2}{2} = a + ap^2$

$p = \dfrac{x}{a} \quad \therefore \; y = a + a\left(\dfrac{x^2}{a^2}\right)$

$$y - a = \frac{x^2}{a}$$

or $x^2 = a(y-a)$ with $x=0$ excluded.

This is a parabola with vertex $(0, a)$ + focal length $\frac{a}{4}$. However, vertex $(0, a)$ must be excluded from locus.

d) $\dfrac{dM}{dt} = k(S-M)$

i) $M = S + Ae^{-kt}$

$\dfrac{dM}{dt} = 0 + Ae^{-kt} \cdot -k$

$= -kAe^{-kt}$

But $M - S = Ae^{-kt}$

$\therefore \dfrac{dM}{dt} = -k(M-S)$

$= k(S-M)$ as required.

ii) $t=0$, $M=0$

$M = S + Ae^{-kt}$

$0 = S + A \Rightarrow A = -S$

$t=10$, $M=40$, $S = 100$ (%)

$40 = 100 - 100e^{-10k}$

$100e^{-10k} = 60$

$e^{-10k} = \dfrac{3}{5} \Longrightarrow e^{10k} = \dfrac{5}{3}$

$\therefore 10k = \ln 5/3$

$k = \dfrac{1}{10} \ln \dfrac{5}{3}$ as required

$\therefore M = S + Ae^{-\frac{1}{10}\ln\frac{5}{3}t}$

13 a) $\frac{d^2x}{dt^2} = -4x$

i) $x = a\cos(2t+\beta)$

$\frac{dx}{dt} = v = -2a\sin(2t+\beta)$

$\frac{d^2x}{dt^2} = \ddot{x} = -4a\cos(2t+\beta)$

$= -4x$ as required

period $= \frac{2\pi}{2} = \pi$

ii) $t=0,\ v=2,\ x=4$

$4 = a\cos\beta \implies \cos\beta = \frac{4}{a}$

$2 = -2a\sin\beta \implies \sin\beta = -\frac{1}{a}$

$\sin^2\beta + \cos^2\beta = 1$

$\frac{1}{a^2} + \frac{16}{a^2} = 1$

$\frac{17}{a^2} = 1 \implies a = \sqrt{17}\ (a>0)$

iii) $\tan\beta = \frac{-1/a}{4/a} = -\frac{1}{4}$

cos +ve sin −ve $\therefore$ 4th quad for β.

$\beta = \tan^{-1}\left(-\frac{1}{4}\right) \approx -0.245$

iv) $x=4$ $x = \sqrt{17}\cos(2t-0.245)$

$4 = \sqrt{17}\cos(2t-0.245)$

$\cos(2t-0.245) = \frac{4}{\sqrt{17}}$

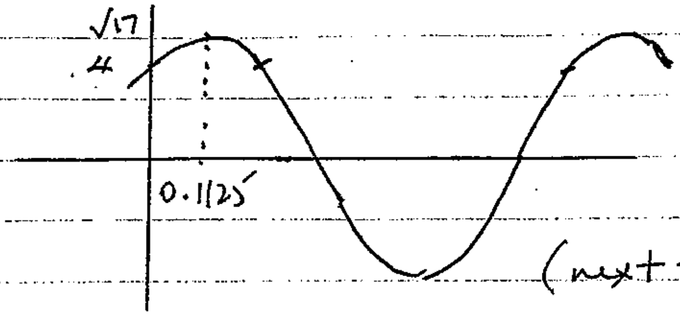
$2t - 0.245 = 2n\pi \pm \cos^{-1}\frac{4}{\sqrt{17}}$

$2t - 0.245 = 2n\pi \pm 0.245$

$2t = 2n\pi + 2\times 0.245$ or $2n\pi$

$t = n\pi + 0.245$ or $n\pi$

v) $n=0$, $t=-\beta = 0.245$



$x = \sqrt{17}\cos(2t - 0.245)$

1st time is 0.245 secs
(next time is π)

b) $$\int_3^8 \frac{x-1}{\sqrt{x+1}}\,dx$$

$u = \sqrt{x+1}$

$u^2 = x+1 \qquad x = u^2 - 1$

$2u\,du = dx$

$x=3, \quad u=2$

$x=8, \quad u=3$

$$= \int_2^3 \frac{u^2-2}{u} . 2u\,du$$

$$= 2\int_2^3 u^2 - 2\,du \quad = 2\left[\frac{u^3}{3} - 2u\right]_2^3$$

$$= 2\left(\frac{27}{3} - 6 - \frac{8}{3} + 4\right)$$

$$= 8\tfrac{2}{3}$$

c) $P(x) = x(x+1) - a(a+1)$

$P(a) = a(a+1) - a(a+1)$

$= 0$

$\therefore$ $x-a$ is a factor.

Otherwise method:

$x^2 + x - a^2 - a$

$(x^2 - a^2) + (x-a)$

$(x-a)(x+a) + (x-a)$

$P(x) = (x-a)(x+a+1)$

d) $f(x) = e^{-x^2} - 5x^2 - 0.99$

$f'(x) = e^{-x^2} \cdot -2x - 10x$

$= -2x e^{-x^2} - 10x$

$= -2x \{e^{-x^2} + 5\}$

$f(0) = 1 - 0.99 = 0.01$

$f'(0) = 0$

$x_1 = x_0 - \frac{f(0)}{f'(0)}$

$= 0 - \frac{0.01}{0}$

Method fails because there is a stationary point at $x=0$.
$\therefore$ Tangent at $x=0$ does not cross x axis for next approximation.

Q 14 $\quad \frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2} - \cdots - \frac{1}{x^n} = \frac{1}{x^n(x-1)}$

① Prove true for $n=1$

$LHS = \frac{1}{x-1} - \frac{1}{x} = \frac{x-(x-1)}{x(x-1)} = \frac{1}{x(x-1)}$

$RHS = \frac{1}{x^1(x-1)}$ as required

$\therefore$ true for $n=1$

② Assume true for $n=k$

ie $\frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2} - \cdots - \frac{1}{x^k} = \frac{1}{x^k(x-1)}$ $\quad *$

③ Consider $\frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2} \cdots - \frac{1}{x^k} - \frac{1}{x^{k+1}}$

$= \frac{1}{x^k(x-1)} - \frac{1}{x^{k+1}}$ using $*$

$= \frac{x-(x-1)}{x^{k+1}(x-1)} = \frac{1}{x^{k+1}(x-1)}$

which is of the form $\frac{1}{x^n(x-1)}$ when

$n = k+1$.

(4) Since statement is true for $n=1$ and if true for $n=k$, is also true for $n=k+1$ then it is true for all positive integers n.

b) $3x^3 + 8x^2 - 1 = 0$

i) $\Sigma\alpha = \alpha+\beta+\gamma = -\frac{8}{3}$ $\left(-\frac{b}{a}\right)$

$\Sigma\alpha\beta = \alpha\beta + \beta\gamma + \gamma\alpha = 0$ $\left(\frac{c}{a}\right)$

$\alpha\beta\gamma = \frac{-d}{a} = \frac{1}{3}$

ii) $\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\gamma}\right)\left(\gamma+\frac{1}{\alpha}\right) =$

$\alpha\beta\gamma + \alpha\beta\cdot\frac{1}{\alpha} + \alpha\frac{1}{\gamma}\cdot\gamma + \alpha\frac{1}{\gamma}\frac{1}{\alpha} + \frac{1}{\beta}\beta\gamma +$

$\frac{1}{\beta}\cdot\beta\cdot\frac{1}{\alpha} + \frac{1}{\beta}\cdot\frac{1}{\gamma}\cdot\gamma + \frac{1}{\beta}\cdot\frac{1}{\gamma}\cdot\frac{1}{\alpha}$

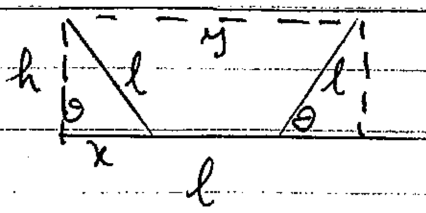
$= \alpha\beta\gamma + (\alpha+\beta+\gamma) + \left(\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}\right) + \frac{1}{\alpha\beta\gamma}$

$= \alpha\beta\gamma + (\alpha+\beta+\gamma) + \frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma} + \frac{1}{\alpha\beta\gamma}$

$= \frac{1}{3} - \frac{8}{3} + 0 + 3 = \frac{2}{3}$

c) $\frac{h}{l} = \sin\theta$

$h = l\sin\theta$



$\frac{x}{l} = \cos\theta$

$x = l\cos\theta$

$\therefore y = 2x + l = 2l\cos\theta + l$

$$\text{Area} = \frac{h}{2}(a+b) = \frac{l\sin\theta}{2}\left(2l\cos\theta + l + l\right)$$

$$\text{Area} = \frac{l\sin\theta}{2}\left\{2l\cos\theta + 2l\right\}$$

$$A = l^2\sin\theta(\cos\theta + 1) \quad \text{as required}$$

ii) $$\frac{dA}{d\theta} = l^2\left\{\sin\theta(-\sin\theta) + (\cos\theta+1)\cos\theta\right\}$$

$$= l^2\left\{-\sin^2\theta + \cos^2\theta + \cos\theta\right\}$$

$$= l^2\left\{\cos^2\theta - 1 + \cos^2\theta + \cos\theta\right\}$$

$$= l^2\left\{2\cos^2\theta + \cos\theta - 1\right\}$$

$\frac{dA}{d\theta} = 0$ for maximum or minimum area.

$$2\cos^2\theta + \cos\theta - 1 = 0$$

$$(2\cos\theta - 1)(\cos\theta + 1) = 0$$

$\cos\theta = \frac{1}{2}$ or $\cos\theta = -1$

$\theta = \frac{\pi}{3}$ $\qquad$ out of range of θ

Show that Area is infact maximum.

$$\frac{d^2A}{d\theta^2} = l^2\left\{4\cos\theta(-\sin\theta) - \sin\theta\right\}$$

$$= -l^2\left\{4\cos\theta\sin\theta + \sin\theta\right\}$$

when $\theta = \frac{\pi}{3}$, $4\cos\theta\sin\theta + \sin\theta$ is positive

$\therefore \frac{d^2A}{d\theta^2}$ is negative

$\therefore A$ concave down

$\therefore$ Area is maximum.

$\therefore$ Area is maximum when $\theta = \pi/3$